**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans – Given, *μ*=45 min

*σ* = 8 min

X=50

Z = (X-45)/8

= (50-45)/8

= 0.625

P(Z ≤0.625) = 0.7324

P(not commit)= 1- 0.7324

= 0.2676

**B. = 0.2676**

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

Ans -  **False ,**

( z-score for 44 = (44 - 38)/6 = 1(z value from table is 0.84134) = 84.13 %

People above 44 age = 100 - 84.13 = 15.87% ≈ 63

 out of 400)

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1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans - **True,**

z-score (Employee=30) = (30-38)/6 = -1.333(z value from table is 0.09176)

P(Employee=30) = 0.09176

No. of Employees attending training Program =

=400\*0.09176

= 36.706

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans - As both are independent normal random variables, X1 + X2 is normal with

N(µ1+µ2,σ12 +σ22 ).

And 2X1 will just scale the normal distribution by 2 times.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans - x=σ[z] + µ

Thus “a” = 0.5th percentile for X = 20\*[-2.57] + 100 = 48.5

& ”b” = 99.5th percentile for X = 20\*[+2.57] + 100 = 151.4

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Ans - 95% of the **probability lies**between 1.96 **standard deviations**of the **mean.**The Range is

= (12-1.96 \* 5.12+ 1.96\*5)

= ($ 2.2M, $ 22.8M)

=(Rs. 99M, Rs. 1026M)

1. Specify the 5th percentile of profit (in Rupees) for the company

Ans - 5th percentile is calculated as :

X̅ = µ + (z5 \* σ)

Z5 = -1.5

= 540 + (-1.5\*225)

=202.5 Million RS.

1. Which of the two divisions has a larger probability of making a loss in a given year?

Ans - Division 2 with distribution N(7, 42)